E-ISSN: 2579-9258
P-ISSN: 2614-3038

Jurnal Cendekia: Jurnal Pendidikan Matematika
Volume 08, Nomor 01, Desember 2023-Maret 2024, pp. 714-724

An Exploration of High School Students' Errors in Solving Trigonometry Problems

Putri Armania Agustina Alfitri[1]✉, Nurdiyah Kurniati [2], Sufyani Prabawanto[3], Kartika Yulianti[4]

[1, 2,3,4] Program Studi Pendidikan Matematika, Universitas Pendidikan Indonesia,
Jl. Dr. Setiabudi No.229, Isola, Kec. Sukasari, Kota Bandung, Jawa Barat, Indonesia
putriarmaniaaa@upi.edu

Abstract

Trigonometry plays an important role in the field of mathematics education, but in fact, some errors are identified. Therefore, this study aims to explore the students' errors in solving trigonometry problems, especially in the sub-topics of trigonometric ratios. This is a qualitative study using a case study design. The subjects of this study were 35 students in grade 10 in one of the senior high schools in a district located in the Province of East Kalimantan. The research instruments were 1) a diagnostic test consisting of five essay problems about trigonometric ratios; 2) observation sheets; and 3) interview guidelines. The data was analyzed through three processes. They were: 1) data reduction, 2) data presentation, and 3) conclusion drawing or verification. The results show that the subjects made errors in solving trigonometric ratio problems, with the most errors made by subjects being errors in understanding concepts and writing algorithms. The causes of students making errors are due to the weak understanding of students' concepts, lack of student mastery of the prerequisite topics, the problems given being less varied, and students being in a hurry to solve problems.
Keywords: error, trigonometry, problems

Abstrak

Trigonometri memiliki peran yang penting di bidang pendidikan matematika, namun pada faktanya masih terdapat kesalahan-kesalahan yang teridentifikasi. Oleh karena itu, penelitian ini bertujuan untuk mengeksplorasi kesalahan-kesalahan yang dilakukan siswa dalam menyelesaikan masalah trigonometri, khususnya pada sub materi rasio trigonometri. Penelitian ini menggunakan metode kualitatif dengan desain studi kasus. Subjek penelitian ini adalah 35 siswa kelas X di salah satu SMAN di Kabupaten Kutai Kartanegara, Kalimantan Timur. Pengumpulan data melalui observasi, tes dan wawancara. Data yang telah terkumpul dianalisis melalui tiga proses yaitu reduksi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa kesalahan dalam menyelesaikan masalah rasio trigonometri ditemukan pada setiap subjek penelitian, dengan kesalahan terbanyak yang dilakukan siswa adalah kesalahan dalam memahami konsep dan kesalahan dalam menuliskan algoritma. Penyebab siswa melakukan kesalahan adalah karena lemahnya pemahaman konsep siswa, kurangnya penguasaan siswa terhadap materi prasyarat, masalah yang diberikan
Kata kunci: kesalahan, trigonometri, masalah

Copyright (c) 2024 Putri Armania Agustina Alfitri, Nurdiyah Kurniati, Sufyani Prabawanto, Kartika Yulianti

✉ Corresponding author: Putri Armania Agustina Alfitri
Tri Wulandari Email Address: putriarmaniaaa@upi.edu (Jl. Dr. Setiabudi Kota Bandung, Jawa Barat, Indonesia)
Received 06 January 2023, Accepted 04 January 2024, Published 31 March 2024
DoI: https://doi.org/10.31004/cendekia.v8i1.2109

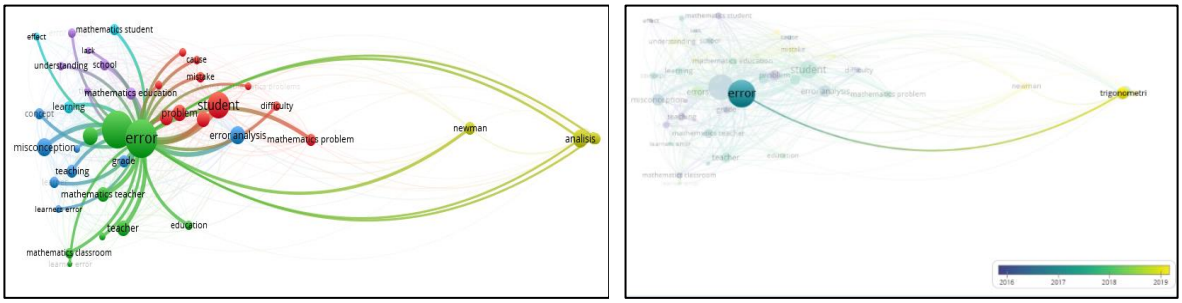
INTRODUCTION

Trigonometry plays an important role in the field of mathematics education. This is in line with the statement that trigonometry is an essential topic and can not be separated from mathematics at the high school level (Huda & Qohar, 2021; Jelatu et al., 2019; Kamber & Takaci, 2018; Yang & Sianturi, 2017). In addition, Weber stated that trigonometry is one of the basic mathematics topics that form the basis for further topics such as algebra, geometry, graphic reasoning, pre-calculus, and calculus Weber (2005). On the other hand, trigonometry also plays a role outside the field of mathematics education, namely contributing to developing science and technology and in everyday life such as measuring the height of buildings, flight engineering, helping to create a robot, and playing a role in the health sector

(Emmanouil et al., 2016; Goel, 2015; Jelatu et al., 2019; Weber, 2005; Yang & Sianturi, 2017). Therefore, trigonometry is one of the essential topics in learning mathematics.

Although trigonometry is an essential topic. In practice, trigonometry is often assumed to be a very difficult topic (Kamber & Takaci, 2018; Walsh et al., 2017) so that the errors experienced by students are identified. According to the relevant research discovered by Rohimah that students make errors because they have difficulty understanding angles related to trigonometry, and in calculating or operating trigonometric problems Rohimah and Prabawanto (2019). More findings by Usman, namely errors made by students including understanding, transformation errors, and process skill errors Usman and Hussaini (2017). As well as errors identified as a result of students' lack of mastery of prerequisite topics (Brijlall & Niranjan, 2015). Students' errors are caused by several factors, such as internal and external factors. The internal factors refer to an incomplete understanding of students' concepts, and the external factors refer to learning resources that students receive (Brijlall & Niranjan, 2015; Chigonga, 2016; Rohimah & Prabawanto, 2019). Thus, based on the results of previous studies, errors or misconceptions of students have been identified in solving problems on trigonometry topics.

Furthermore, based on the findings of previous studies in the last 10 years, it was found that the visualization of research trends from bibliometric results for the scope of students' errors or misconceptions in solving math problems, there is still little research to discuss trigonometry topics but become a research trend in 2019 to on. The display of the analysis results in bibliometrics can be seen in the following Figure 1.



(a) (b)

Figure 1. Bibliometric analysis: (a) the results of the network visualization analysis and (b) the results of the overlay visualization analysis

Based on the facts on the importance of trigonometry topics, the identification of student errors in solving trigonometry problems, and the research gap, this study aims to explore the errors made by students in solving trigonometry problems, especially in the sub-topics of trigonometric ratios as the basic topics in trigonometry.

The types of errors used in this study are modifikasi dari teori Ashlock (2006): 1) Error in understanding the concept; 2) error in setting operation; 3) error in performing calculations; (4) error in writing or applying principles; 5) errors in writing algorithms; and 6) random response. Based on the types of errors, the indicators of the types are presented in the following table 1.

Table 1. Error Types Indicator

Error Types	Indicators
Error in understanding concepts	Students misunderstand or apply a definition in the matter of trigonometric ratios; students misunderstood the problem given
Error in setting operation	Students are not right in determining an operation in the steps of problem-solving
Error in performing the calculation	Students are correct in applying the operation but wrong in determining the final result or the value of the operation
Error in applying the principle	Students are not precise in writing or using formulas, theorems, or rules in solving problems
Error in writing algorithm	Students are correct in determining the final result but there are errors in the completion steps; students do not write down the final answer asked about the question; the problem-solving strategy that students choose is not right
Random Response	There is no relationship between the completion steps and the problems given or the answers written by students carelessly are not related to the problems given.

METHODS

The type of this study was qualitative with a case study design. The subjects of this study were 35 students in grade 10 in one of the senior high schools in a district located in the Province of East Kalimantan. Subject selection is based on various categories of students' abilities such as low, medium, and high categories. The research instruments were 1) a diagnostic test consisting of five essay problems about trigonometric ratios; 2) observation sheets; and 3) interview guidelines. The validity of the data was carried out by the triangulation technique which is a combination of test data, observations, and interviews.

The feasibility test for diagnostic tests is carried out through validation by experts in the field of mathematics, consisting of two graduates of master's students in mathematics education and one teacher in the field of study. The following is a description of the results of the feasibility test analysis of 3 aspects: content, construction, and language.

The content aspect shows that the suitability of the question items with the learning objectives, learning materials, and question indicators has been fulfilled. So that the content aspect can be declared valid. Furthermore, the construction aspect shows that the main questions are formulated briefly and clearly, and the question items do not lead to the answer key. So that the construction aspect can be declared feasible or valid. Finally, in the language aspect, it can be seen that the use of words meets PUBI and each word's meaning does not contain ambiguity. So that the language aspect can be declared valid.

Test and observation data were used to identify student errors in solving trigonometric ratio problems, while the interview data were used to clarify errors obtained from test and observation data

and to identify the causes of errors made by students.

The data analyzed based on the interactive model of analysis promoted by Miles and Huberman (1994), namely: 1) data reduction, namely the researcher reduces the information that is not needed in the study based on predetermined error indicators; 2) data presentation, namely the researcher presents data in the form of pictures and narratives in order to find out in detail the identified errors; and 3) drawing conclusions/verification, namely the configuration of the results of reduction and presentation data so as to obtain meaning from the research results (Miles & Huberman, 1994).

RESULTS AND DISCUSSION

This study aims to explore the errors made by students in solving trigonometric ratio problems. The subjects of this study were 35 students in grade 10 in one of the senior high schools in a district located in the Province of East Kalimantan. The errors identified by the research subjects are presented in the following table 2.

Table 2. Classification of Students' Errors

No.	Error Type	Question Number					Total	%
		1	2	3	4	5		
1	Error in Understanding Concept	24	13	23	23	23	106	30
2	Error in setting operation	0	0	4	2	10	16	5
3	Error in performing calculation	12	8	5	6	13	44	13
4	Error in applying principle	5	8	3	9	8	33	9
5	Error in writing algorithm	30	24	3	25	24	106	30
6	Random Response	10	14	15	3	5	47	13
Total		**81**	**67**	**53**	**68**	**83**	**352**	**100**

Tab. 2 shows the Classification of Students' Errors. It can be concluded that the most errors made by students are errors in understanding concepts and errors in writing algorithms, which are equal to 30%. While the error that the least students make is an error in setting the operation, which is equal to 5%. This is in line with the findings of Gradini et al (2022) that the most student errors in solving trigonometric problems are conceptual errors with the indicator of student error in understanding the problem, which is equal to 57.73% (Gradini et al., 2022).

From the classification results in Table 2, the researcher selected respondents to obtain more in-depth information, with the following criteria: 1) the student solved the problem on the written test given; 2) the student is dominant in making an error in the given problem, and 3) based on the advice of the mathematics teacher concerned. There were six students chosen as respondents in this study, they were A, CP, DKA, GAP, JKN, and OCP. The following explanation is an exploration of student errors in solving trigonometric ratio problems.

Error in Understanding the Concept

Based on the students' answers, errors in understanding the concept occurred when they wrote

the concepts of trigonometric ratios. The student's answer is as follows.

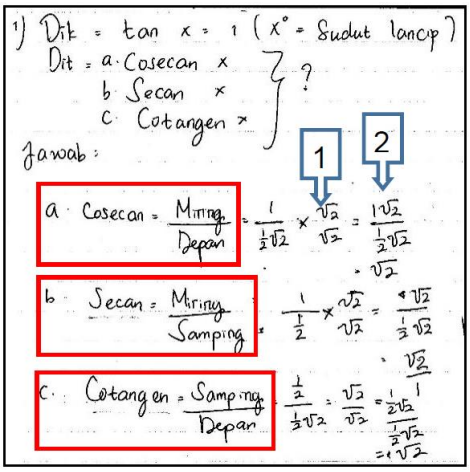


Figure 2. OCP's Answer to Error in Understanding Concept

R : Next, Check your answer first. Do you think the steps you are taking are correct? (Student points to the wrong answer).

OCP : In my opinion, this is true ka.

R : And then, what's mean about your writing? Do you think there is still something wrong?

OCP : *Oh yeah, that is cosecan x I mean.*

Fig. 2 (marked in red) shows that OCP was writing the concepts of trigonometric ratios wrongly. OCP did not write the angles for every trigonometric function. In addition, OCP applied the concepts of trigonometric ratios in a mnemonic strategy, namely "front as side opposite, oblique as hypotenuse, and side as side adjacent".

Error in Setting Operation

Based on the students' answers, errors in setting operations occurred when they determined math operations to solve the problems of trigonometric ratios. The student's answer is as follows.

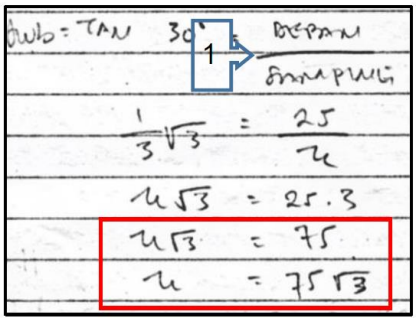


Figure 3. DKA's Answer to Error in Setting Operation

R : Do you know what the right answer?

DKA : Actually, here I'm so confused ka. What is $x\sqrt{3} = 75$ mean?

R : Confused or dont understand?

DKA : I don't understand. When I answer the question and the time is almost over. So, I just write directly.

Fig. 3 (marked in red) shows that DKA made an error when she determined the value of x. The DKA's process in determining the value is x by dividing each of the two sides by $\sqrt{3}$, but the result is $75\sqrt{3}$. It should be $\frac{75}{\sqrt{3}}$.

Errors in Performing Calculation

Based on the students' answers, errors in performing calculations occurred when they determined the result of the addition operation. Furthermore, the students made errors when they rationalized denominators of radical expressions. The student's answers are as follows.

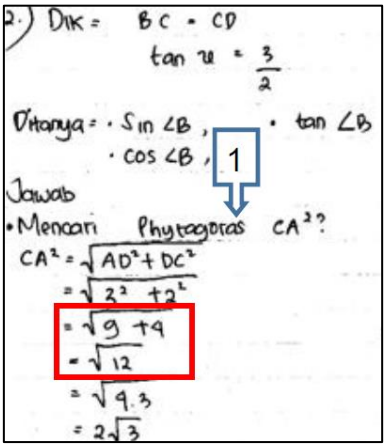


Figure 4. CP's Answer to Error in Performing Calculation

R : Allright, and then why $\sqrt{9+4} = \sqrt{12}$? Are you calculatios this correctly?

CP : Of course!

R : Are you sure? Can you recalculate?

CP : Student start to calculate and say 13.

R : Yeah, why you can't calculate the answer correctly?

CP : *Miscalculated ka, I calculated too fast.*

Fig. 4 (marked in red) shows that CP made an error in the calculation results of $\sqrt{9+4}$ that student answered $\sqrt{12}$. The result should be $\sqrt{13}$.

Errors in Applying the Principle

Based on the students' answers, errors in applying principles are applied to the principles of the Pythagorean theorem and rationalized denominators of radical expressions. The student's answers are as follows.

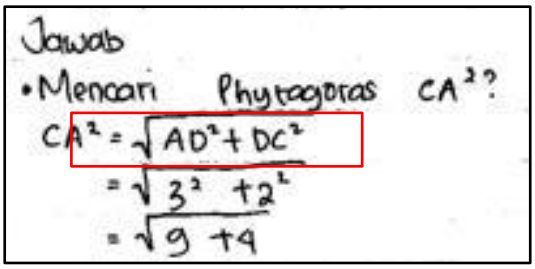


Figure 5. GAP's Answer to Error in Applying the Principle

R : *Okay, let me know about the Pythagorean theorem looking for length of CA?*

GAP : $CD^2 + DA^2$

R : Correct. Then, your answer is $CA^2 = \sqrt{CD^2 + DA^2}$. Does CA need to be a power of two?

GAP : No

R : So, Why your answer is CA^2?

GAP : I want to finish it quickly, kak.

Fig.5 (marked in red) shows that CP understood the application of the Pythagorean theorem, but was not careful in writing the Pythagorean theorem, namely, CP wrote $\mathrm{CA}^2 = \sqrt{\mathrm{AD}^2 + \mathrm{DC}^2}$. The theorem should be $\mathrm{CA}^2 = \mathrm{AD}^2 + \mathrm{DC}^2$.

Errors in Writing Algorithm

Based on the students' answers, errors in writing algorithms are the solution strategy chosen by the student was not compatible, the student completion step was not clear, and the student's final answer did not match the problem. The student's answers are as follows.

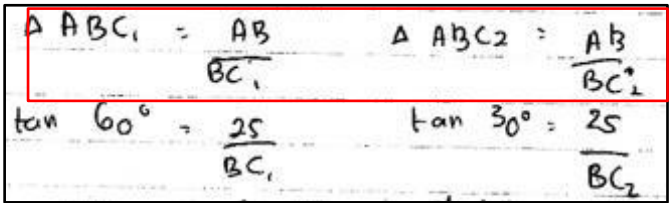


Figure 6. JKN's Answer to Errors in Writing Algorithms

JKN : *Yesterday, this scribble was so long kak. (students pointed to the answer at number 4). I didn't find it, then immediately started working on number 5, number 5 was also in a hurry, didn't just use scribbles straight away.*

R : Allright. Because that is your job, there are a lot of mistake, right? So, try to make sure your steps to rationalize what you wrote! Do you think the steps are correct?

JKN : Oh I see, It's Wrong.

R : Why?

JKN : I'm in a hurry, kak.

Fig. 6 (marked in red) shows that JKN was aware of choosing the right strategy to solve the problem. However, the solution steps that JKN wrote were incorrect. JKN intended to use $\Delta\mathrm{ABC}_1$ and $\Delta\mathrm{ABC}_2$ to apply trigonometric ratios, but JKN wrote $\Delta\mathrm{ABC}_1 = \frac{\mathrm{AB}}{\mathrm{BC}_1}$ and $\Delta\mathrm{ABC}_2 = \frac{\mathrm{AB}}{\mathrm{BC}_2}$. It is not compatible with scientific conception.

Random Response

Based on the students' answers, random response occurred when the student's answer was not compatible with the problem. The student's answer is as follows.

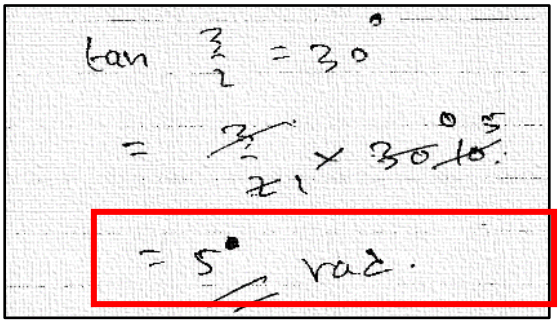


Figure 7. A's answer to Random Response

R : See, your answer is $tan\frac{3}{3} = 30°$! Why radians appeasrs on your answer?

A : *Because, Tan that i know like yesterday. In my logic, I only know that rad is* 30°. *So, I write* $\frac{3}{2} \times 30°$, *And then I ask to my friend if this question has radians or not, and my friend said if that is radians.*

R : *Well, logically how can you say if* $tan\frac{3}{2} = 30°$*?*

A : I remember just like that.

R : Allright, So $tan\ \frac{3}{2}$ not 30°. So, your logic is wrong.

A : *Yeah, because I only study about rad. So, I only understand about that.*

Fig. 7 shows that A's answer did not match the problem information because the information does not mention radians. In addition, A was not careful in calculating the value of $\tan\frac{3}{2}$ which is answered $\tan\frac{3}{2} = 30°$.

Based on the errors profile and analysis of errors, researchers conducted interviews to clarify respondents' answers and identify the causes of students making the errors. Based on the results of the interviews obtained the following information.

The cause of students making errors in understanding the concept is because there is no knowledge construction process that students receive during learning, resulting in a weak understanding of students' concepts. In addition, the mnemonic strategy that the teacher applies causes students not to remember the actual concept so that when answering the problem students use the memory instead of using the scientific conception. This is in line with the findings of Nanmumpuni & Retnawati that students wrote the concept of trigonometric ratios using the words "front and side" Nanmumpuni and Retnawati (2021). In addition, Dewanto et al (2017) also found that students wrote down the concept of trigonometric ratios in "de, sa, and mi".

The lack of student mastery of the prerequisite topics is the dominant cause of students making errors, namely errors in determining operations, errors in performing calculations, and errors in

applying principles. The prerequisite topics in question are the topics for dividing fractions, determining the solution of linear equations in one variable, multiplying radical expression, rationalizing the denominator of radical expression, and the Pythagorean theorem. This is in line with the findings of Maharani & Bernard that students understood the strategies for solving problems but they made errors because they are still low in calculations, this is due to a lack of knowledge of the prerequisite topics in students Maharani and Bernard (2018).

The cause of students making mistakes in writing algorithms is because students are not used to dealing with application problems or math problems. In addition, the cause of students making mistakes in writing algorithms is because students are in a hurry to solve problems so students are not careful. This is in line with Setiawan (2021) that students make mistakes in applying trigonometric ratios to right triangles because they are not careful and do not re-check the answers obtained.

The cause of students' random responses is because students do not know the strategies that must be used in solving problems, students do not want the answers to be empty, so students answer carelessly, and students only remember certain topics so they answer according to what is remembered even though it is not in accordance with the information on the problem.

CONCLUSION

Based on the results and discussion, it can be concluded that the students' errors in solving trigonometric ratio problems are: 1) errors in understanding the concept, namely the students' inaccuracy in writing the concept of trigonometric ratios; 2) errors in setting operations, namely students incorrectly determining operations to the solution steps; 3) errors in performing calculations, namely the results of addition and multiplication operations in the form of roots; 4) errors in applying the principle, namely the students' inaccuracy in writing the Pythagorean theorem and rationalizing the denominator of the root form; 5) errors in writing algorithms, namely the strategies and steps of student completion are not appropriate and the final answer is not in accordance with the problem; and 6) random response, i.e. there is no relationship between students' answers and the problems given. The causes of students making mistakes are due to a weak understanding of students' concepts, lack of student mastery of the prerequisite topics, the problems are given less varied, and students being in a hurry to solve problems. Based on the research that has been done, the recommendation for further researchers is that the results of this study are expected to be a guide in preparing a learning design, which can lead to minimizing student errors in solving trigonometric ratio problems and students rush in solving problems.

REFERENCES

Ashlock, R. B. (2006). *Error Patterns in Computation.* Pearson Prentice Hall.

Brijlall, D., & Niranjan, C. (2015). Using Manipulatives to Support an Embodied Approach to Learning Trigonometry in A South African School: A Case Study. *Africa Education Review*,

12(3), 361–380. https://doi.org/10.1080/18146627.2015.1110893

Chigonga, B. (2016). Learners' Errors When Solving Trigonometric Equations and Suggested Interventions From Grade 12 Mathematics Teachers. *International Society for Technology in Education*, 163–176. https://www.researchgate.net/publication/314285050_Learners'_errors_when_solving_trigonometric_equations_and_suggested_interventions_from_Grade_12_Mathematics_teachers

Dewanto, M. D., Budiyono, B., & Pratiwi, H. (2017). Students' Error Analysis in Solving The Math Word Problems of High Order Thinking Skills (HOTS) Type on Trigonometry Application. *International Conference on Science, Mathematics, and Education*, *218*, 195–200. https://doi.org/10.2991/icomse-17.2018.34

Emmanouil, E., Wei, G., & Dai, J. S. (2016). Spherical Trigonometry Constrained Kinematics For a Dexterous Robotic Hand With an Articulated Palm. *Robotica*, *34*(12), 2788–2805. https://doi.org/10.1017/S0263574715000399

Goel, S. (2015). Reform of Teaching a Trigonometry Course. *Georgia Journal of Science*, *73*(2), 136–146. https://digitalcommons.gaacademy.org/gjs/vol73/iss2/3%0AThis

Gradini, E., Yustinaningrum, B., & Safitri, D. (2022). Kesalahan Siswa Dalam Memecahkan Masalah Trigonometri Ditinjau dari Indikator Polya. *Mushorafa: Jurnal Pendidikan Matematika*, *11*(1), 49–60. http://journal.institutpendidikan.ac.id/index.php/mosharafa

Huda, R., & Qohar, A. (2021). Student Activeness and Understanding in Mathematics Learning Using GeoGebra Application on The Trigonometry Ratio Topic. *AIP Conference Proceedings*, *2330*(040034), 1–8. https://doi.org/10.1063/5.0043140

Jelatu, S., Kurniawan, Y., Kurnila, V. S., Mandur, K., & Jundu, R. (2019). Collaboration TPS Learning Model and M-Learning Based on Android for Understanding of Trigonometry Concepts With Different Cognitive Style. *International Journal of Instruction*, *12*(4), 545–560. https://doi.org/10.29333/iji.2019.12435a

Kamber, D., & Takaci, D. (2018). On Problematic Aspects in Learning Trigonometry. *International Journal of Mathematical Education in Science and Technology*, *49*(2), 161–175. https://doi.org/10.1080/0020739X.2017.1357846

Maharani, S., & Bernard, M. (2018). Analisis Hubungan Resiliensi Matematik Terhadap Kemampuan Pemecahan Masalah Siswa Pada Materi Lingkaran. *JPMI (Jurnal Pembelajaran Matematika Inovatif)*, *1*(5), 819. https://doi.org/10.22460/jpmi.v1i5.p819-826

Miles, M. B., & Huberman, A. M. (1994). *Qualitative Data Analysis: An Expanded Sourcebook* (2nd ed.). Sage Publications, Inc.

Nanmumpuni, H. P., & Retnawati, H. (2021). Analysis of Senior High School Student's Difficulty in Resolving Trigonometry Conceptual Problems. *Journal of Physics: Conference Series*, *1776*(1), 1–8. https://doi.org/10.1088/1742-6596/1776/1/012012

Rohimah, S. M., & Prabawanto, S. (2019). Student's Difficulty Identification in Completing the

Problem of Equation and Trigonometry Identities. *International Journal of Trends in Mathematics Education Research*, *2*(1), 34. https://doi.org/10.33122/ijtmer.v2i1.50

Setiawan, Y. E. (2021). Analisis Kesalahan Mahasiswa Semester Pertama dalam Menentukan Nilai Fungsi Trigonometri Sudut Istimewa. *SJME (Supremum Journal of Mathematics Education)*, *5*(1), 110–121. https://doi.org/10.35706/sjme.v5i1.4531

Usman, M. H., & Hussaini, M. M. (2017). Analysis of Students' Error in Learning of Trigonometry Among Senior Secondary School Students in Zaria Metropolis, Nigeria. *IOSR Journal of Mathematics*, *13*(02), 01–04. https://doi.org/10.9790/5728-1302040104

Walsh, R., Fitzmaurice, O., & O'Donoghue, J. (2017). What Subject Matter Knowledge do Second-level Teachers Need to Know to Teach Trigonometry? An Exploration and Case Study. *Irish Educational Studies*, *36*(3), 273–306. https://doi.org/10.1080/03323315.2017.1327361

Weber, K. (2005). Students' Understanding of Trigonometric Functions. *Mathematics Education Research Journal*, *17*(3), 91–112. https://doi.org/10.1007/BF03217423

Yang, D. C., & Sianturi, I. A. (2017). An Analysis of Singaporean Versus Indonesian Textbooks Based on Trigonometry Content. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(7), 3829–3848. https://doi.org/10.12973/eurasia.2017.00760a